Circuits II

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1 Time-Domain Response of Circuits

Lecture 1

2026-01-05

We will study the time-domain response of RL, RC, and RLC circuits.

1.1 RC and RL circuits

We first look at first-order circuits.

Definition 1. A capacitor is a device that stores energy in the form of an electric field. The *capacitance* (in Farads F) is a measure of the ability to store energy arising from stored charge.

$$C = \frac{q}{V}.$$

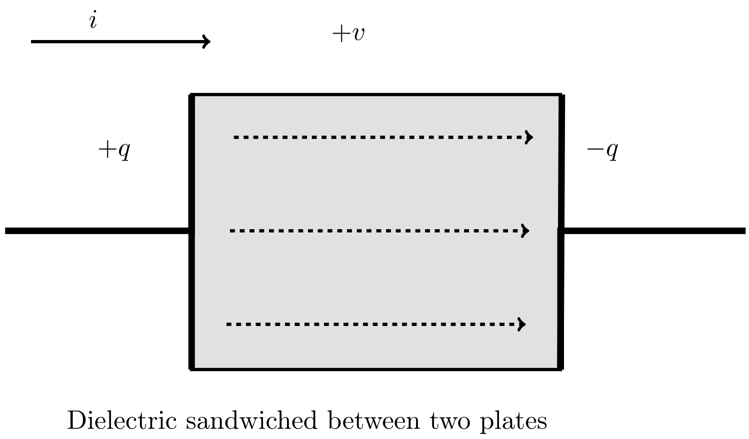

Figure 1: A capacitor with two metallic plates.

We always label current going into the $+v$ terminal to obey passive sign convention. We sometimes label a capacitor with a curved plate to indicate the negative terminal.

2026-01-07 Lecture 2

The magnitude of stored charge on a capacitor is:

$$q = Cv.$$

Where $q(C)$ is the magnitude of the charge on each plate as a function of time, $C(F)$ is the capacitance, and $v(V)$ is the voltage across the capacitor as a function of time. We sometimes suppress the time notation, but it is implied!

Differentiating with respect to time yields the current through a capacitor:

$$\frac{dq}{dt} = i(t) = C\frac{dv}{dt}.$$

Integrating with respect to time yields:

$$dv = \frac{1}{C}idt \implies \int_{t_0}^{t} dv = \frac{1}{C}\int_{t_0}^{t} i(t)\, dt \iff \Delta v = \frac{1}{C}\Delta Q.$$

t_0 is the *initial time*, frequently defined to be $t_0 = 0$, marking some initial event like a switch or sensor. This indicates that from t_0 to t, the change in voltage $v\left(t\right) - v\left(t_0\right)$ is proportional to the change in charge.

Definition 2. We can find *instantaneous power*, generally $p = i\left(t\right)v\left(t\right)$, delivered to the capacitor as:

$$p(t) = Cv\frac{dv}{dt}.$$

Integrating power, we can find the total energy stored from t_0:

$$W = \int_{t_0}^{t} p(t)\, dt = \int_{t_0}^{t} Cv\, dv = \frac{1}{2}C\left(v(t)^2 - v\left(t_0\right)^2\right).$$

Or indefinitely find $W(v)$:

$$W = \int Cv\frac{dv}{dt}\, dt = \frac{1}{2}Cv^2 + C.$$

Where plugging in $v = 0$ gives that $C = 0$, since there is no stored energy when there is no voltage across the capacitor.

Aspects of the time response:

From $i = C\frac{dv}{dt}$, we note that there is no current flowing through the capacitor if the voltage is not changing. The capacitor acts like an open circuit under DC conditions.

Note that this implies that the voltage across a capacitor *cannot change discontinuously*, since $\frac{dv}{dt} = \infty \implies i \to \infty$ and $p \to \infty$, which is impossible. Taking $t_0^- < t_0 < t_0^+$ arbitrarily close to t_0, for continuity, $v\left(t_0^-\right) = v\left(t_0\right) = v\left(t_0^+\right)$.

However, the current $i\left(t\right)$ through a capacitor can change discontinuously, or $i\left(t_0^-\right) \neq i\left(t_0^+\right)$.

Definition 3. An inductor is a device that stores energy in the form of a magnetic field. The inductance (in Henries H) is a measure of the ability to store energy arising from current.

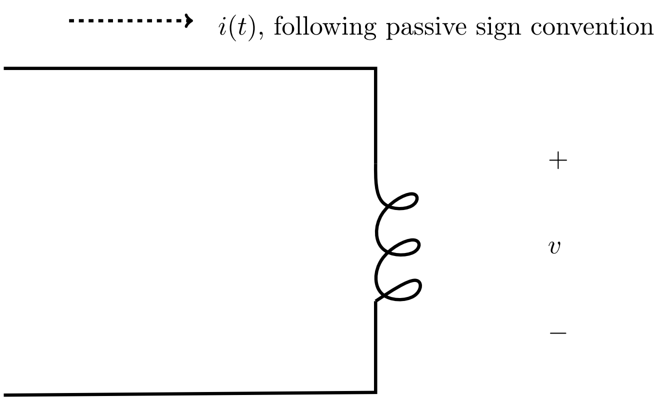

Figure 2: example of an inductor

The total magnetic flux through an inductor is $\phi = Li$.

Definition 4. The voltage across an inductor is

$$v = L\frac{di}{dt}.$$

Integrating, we find

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0).$$

Definition 5. The power through an inductor is:

$$p(t) = Li(t)\frac{di}{dt}.$$

Power can be integrated over time to get the stored energy:

$$W(t) = \frac{1}{2}Li^2.$$

Aspects of the time response:

The voltage across the inductor is 0 when the current is not changing, so it acts like a short circuit under DC conditions.

Additionally, by $p(t)\,Li(t)\,\frac{di}{dt} \implies$ the current cannot change discontinuously. The voltage across an inductor can step, for example in the instant after closing the circuit.

2026-01-08

Lecture 3

1.2 Mathematical first order response

Definition 6. First order circuits involve resistors, sources, and exactly one reactive element, either one reduced capacitor or one reduced inductor. They are governed by a first order differential equation, our current or voltage function has the form:

$$x(t) = x_n(t) + x_f(t).$$

Where x is the complete response, x_n is the homogeneous or natural response, and x_f is the particular or forced response. The natural response (homogeneous solution), when the input is set to 0 (particular solution is 0), has the form:

Note that a network of capacitors or inductors may be simplified into a single *reduced* element, which would qualify for a first order circuit.

Note that any first order circuit, represented by the general form, can reduce the left-hand side of the circuit to a Thevenin or Norton equivalent, which will act on our reactive load on the right side of the circuit. The equivalent portion will contain only i_{eq} or v_{eq}, and R_{eq}.

First, consider a first order circuit with a capacitor from $t > t_0$. The circuit is defined by C, and the Thevenin equivalent v_{eq} and R_{eq}. WLOG, assume passive sign convention is followed.

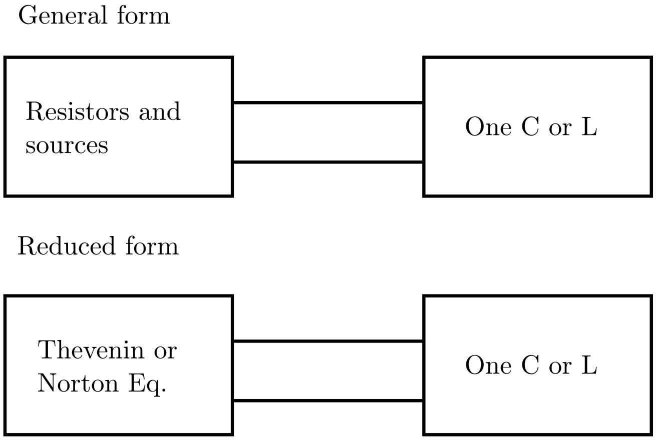

Figure 3: General and reduced form of a first order circuit

Applying KVL:

$$
\begin{aligned}
0 &= v_{eq}(t) - i(t) R_{eq} - v_c(t) \\
0 &= v_{eq} - v_c - R_{eq} C \frac{dv_c}{dt} \\
\frac{v_{eq}}{R_{eq} C} &= \frac{dv_c}{dt} + \frac{1}{R_{eq} C} v_c \\
\int \frac{1}{v_{eq} - v_c} \, dv_c &= \int \frac{1}{R_{eq} C} \, dt \\
v_{eq} - v_c &= A e^{-\frac{t}{R_{eq} C}}.
\end{aligned}
$$

Where $v_c(t)$ is an unknown we seek, $v_{eq}(t)$ is taken to be the known forcing function (source voltage), and R_{eq} and C characterize the circuit. Solving this first order differential equation, we obtain:

$$
v_c = v_{eq} + (v_0 - v_{eq}) e^{-\frac{t}{R_{eq} C}}.
$$

Lecture 4

2026-01-09

Next, consider a first order circuit with an inductor from $t > t_0$. Take an RL circuit with a Norton equivalent source i_{eq} and equivalent resistance R_{eq}, and an inductor L. The single-node voltage drop is v. Take the current through the inductor, following passive sign convention, to be i_L, and similarity, the current through the resistor as i_R. Let us examine $i_L(t)$.

Applying KCL,

$$\begin{aligned} i_{eq} &= i_R + i_L \\ &= \frac{v}{R_{eq}} + i_L \\ &= L\frac{di_L}{dt}\frac{1}{R_{eq}} + i_L. \end{aligned}$$

We can rewrite this as:

$$i_{eq}\frac{R_{eq}}{L} = \frac{di_L}{dt} + \frac{R_{eq}}{L}i_L.$$

With solution:

$$i_L = i_{eq} + Ce^{-\frac{t}{L/R}}.$$

Note. Comparing our two canonical first order differential equations, we notice a general form:

$$\frac{dx\left(t\right)}{dt} + \frac{1}{\tau}x\left(t\right) = \frac{1}{\tau}f\left(t\right).$$

Where $x\left(t\right)$ is the circuit variable of interest, $f\left(t\right)$ is the known forcing function in the circuit, and τ is the characteristic *time constant.*

$$\tau\left(s\right) = \frac{L}{R_{eq}} \text{ or } \tau = R_{eq}C.$$

△

Solving our general form, we can find an integrating factor as:

$$\mu = \exp\left(e^{\frac{t}{\tau}}\right).$$

And apply it as:

$$\begin{aligned} \frac{d}{dt}\left(xe^{\frac{t}{\tau}}\right) &= fe^{\frac{t}{\tau}}\frac{1}{\tau} \\ xe^{\frac{t}{\tau}} &= \frac{1}{\tau}\int fe^{\frac{t}{\tau}}\,dt + A. \end{aligned}$$

Where A is our eventual constant of integration.

$$x\left(t\right) = e^{-\frac{t}{\tau}}\frac{1}{\tau}\int f\left(t\right)e^{\frac{t}{\tau}}\,dt + Ae^{-\frac{t}{\tau}}.$$

We can also approach this by solving the homogeneous equation, and then guessing a form for the particular solution, which will also leave our solution in terms of an integral of $f\left(t\right)$. While seemingly useless, this method of viewing the solution can help understand the intuitive link between the DE and the circuit.

Examine solution from DE lens

The term on the right is $x_n\left(t\right) = Ae^{-\frac{t}{\tau}}$, or the natural response. This form exists even when $f\left(t\right) = 0$, as it is the homogeneous solution to the differential equation. The integral term is the forced response, $x_f\left(t\right) = e^{-\frac{t}{\tau}}\int\frac{f(t)}{\tau}e^{\frac{t}{\tau}}\,dt$.

Further, examining these pieces, it can be shown that x_f satisfies our initial differential equation since it is the particular solution. This is evident since there are no arbitrary constants in x_f. x_n only satisfies the homogeneous equation.

Examine the physical, or intuitive meaning of solution

We note that $x_n \to 0$ as $t \to \infty$, so x_n is the transient part of the response. However, x_f persists as $t \to \infty$, so it is the steady-state portion response, characterized by the forcing function. Without a forcing function present, the signal will die off, which corresponds with having no particular solution.

Simplify our general solution

For reference, our DE solution is:

$$x\left(t\right) = e^{-\frac{t}{\tau}} \frac{1}{\tau} \int f\left(t\right) e^{\frac{t}{\tau}}\, dt + Ae^{-\frac{t}{\tau}}.$$

We typically assume that our forcing function is a constant, $f\left(t\right) = F$. Evaluating the integral and the initial condition, the solution transforms.

$$x\left(t\right) = F + Ae^{-\frac{t}{\tau}}.$$

Since $x \to F$ as $t \to \infty$, and $x\left(t_0^+\right) = x\left(\infty\right) + Ae^{\frac{-t_0^+}{\tau}}$ we may write

$$x\left(t\right) = x\left(\infty\right) + \left(x\left(t_0^+\right) - x\left(\infty\right)\right) e^{\frac{t_0}{\tau}} e^{-\frac{t}{\tau}}, \quad t > t_0.$$

Since x is continuous, we don't need to worry about our careful limit definition with t_0^+. This only works with capacitor voltage and inductor current. Note that the time response is characterized by the initial value $x\left(t_0^+\right)$, the final value $x\left(\infty\right)$, and the time constant τ.

Note. Do not forget that the following analysis assumes $f\left(t\right) = F$, i.e. there are DC conditions at $t = t_0^-$ and $t \to \infty$. $\triangle$

Lecture 5

2026-01-12

Let's examine a new variable, such as the capacitor current or inductor voltage. We used specific relationships, like $i_c = C\frac{dv_c}{dt}$ and $v_L = L\frac{di_L}{dt}$, to build our general first order solution. However, using the linearity of our circuits, this solution apples to all circuit voltages and currents in a first order circuit.

Definition 7. The time constant, $\tau = \frac{L}{R_{eq}}$ or $\tau = CR_{eq}$, characterises the time behaviour, indicating the time for the response to attain $1 - \frac{1}{e} \approx 0.632$ of its steady-state response. While first-order circuits approach their steady-state value asymptotically, we say $x\left(t\right) \approx x\left(\infty\right)$ for $t = 4\tau$, since at this point, $x\left(t\right) = 0.982x\left(\infty\right)$

Lecture 6

2026-01-14

We can apply our form to charging and discharging circuits. Indirectly, we can resolve our differential equations to obtain our canonical form for v_c or i_L, and then convert between different circuit variables using Ohm's Law or other simple relations.

2026-01-16 **Lecture 7**

Sometimes signals in a circuit will be expressed in terms of a unit step function, which results in a circuit event. This unit step function could be reflected or shifted $u(t - t_0)$, resulting in a step up (not reflected) or a step down at time t_0.

Lastly, it is sometimes convenient to write inductors as current sources and capacitors at voltages sources for $t = t_0^+$. Since these variables are always continuous, their values at $t = t_0^-$ can be applied at t_0^+, allowing us to solve for (one-sided) derivatives, and therefore ICs.

1.3 Second-Order Circuits

Definition 8. A second-order circuit has *two irreducible* energy storing elements.

We seek the complete, homogeneous and particular, response (restricted to the time-domain) of second-order circuits, described by a second order DE.

Parallel RLC

Let's first examine a circuit with a current source $i_s(t)$ driving a resistor R, and inductor L, and a capacitor C, each in parallel. First, we mark the capacitor voltage $v_C(t)$ and inductor current

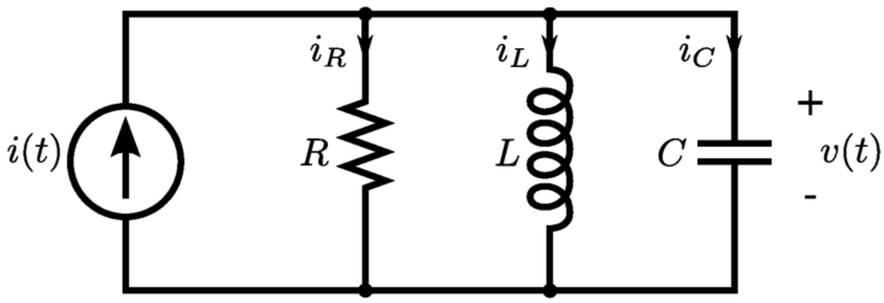

Figure 4: A parallel RLC circuit. Note that our labels may differ from those shown.

$i_L(t)$. Then, we will use circuit laws to develop two first-order DE involving the state variables.

KCL gives:

$$\begin{aligned} i_s &= i_R + i_L + i_C \\ &= \frac{v_c}{R} + i_L + C\frac{dv_c}{dt}. \end{aligned}$$

KVL gives:

$$v_c = L\frac{di_L}{dt}.$$

Combining these two equations, we get:

$$\begin{aligned} i_s &= LC\ddot{i}_L + \frac{L}{R}\dot{i}_L + i_L \\ \frac{i_s}{LC} &= \ddot{i}_L + \frac{1}{RC}\dot{i}_L + \frac{1}{LC}i_L. \end{aligned}$$

Series RLC

Let's first examine a circuit with a voltage source $v_s(t)$ driving a resistor R, and inductor L, and a capacitor C, each in series. Take v_C as the capacitor voltage, and i_L as the inductor current, following passive sign convention. KVL gives:

$$v_s = Ri_L + L\dot{i}_L + v_C.$$

And we note that since the elements are in series:

$$i_L = C\frac{dv_C}{dt}.$$

Combining, we get:

$$\begin{aligned} v_s &= RC\dot{v}_C + LC\ddot{v}_C + v_C \\ \frac{v_s}{LC} &= \ddot{v}_C + \frac{R}{L}\dot{v}_C + \frac{1}{LC}v_C. \end{aligned}$$

Finally, let's first examine a circuit topology with a voltage source $v_s(t)$ driving a series resistor R_1. This connects to a parallel combination of a capacitor C, and a branch with a resistor R_2 in series with an inductor L.

Take v_C as the capacitor voltage, and i_L as the inductor current, following passive sign convention.

First, we note that the voltage drop across the two branches are equal:

$$v_C = i_L R_2 + L\dot{i}_L.$$

We also note that by KCL, the current through R_1 is the sum of the currents through the two branches:

$$\frac{v_s - v_C}{R_1} = i_L + C\dot{v}_C.$$

Combining, we get:

$$\frac{v_s}{R_1} - \frac{i_L R_2}{R_1} - \frac{L\dot{i}_L}{R_1} = i_L + CR_2\dot{i}_L + CL\ddot{i}_L.$$

For a final solution:

$$\ddot{i}_L + \left(\frac{R_2 C + \frac{L}{R_1}}{LC}\right)\dot{i}_L + \left(\frac{1 + \frac{R_2}{R_1}}{LC}\right)i_L = \frac{v_s}{R_1 LC}.$$

Lecture 8

2026-01-19

Definition 9. The general form of the differential equation for the second-order circuit is:

$$\ddot{x} + 2\alpha\dot{x} + \omega_0^2 x = \hat{f}(t).$$

Where $x(t)$ is the time-domain variable of interest, and α and ω_0 are parameters that depend on the circuit elements, and their interconnections. α is the damping coefficient, and ω_0 is the *resonant* frequency.

We examine circuits with $\alpha, \omega_0 > 0$. For our parallel RLC, we found:

- $\alpha = \frac{1}{2RC}$
- $\omega_0 = \frac{q}{\sqrt{LC}}$
- $\hat{f}(t) = \frac{i_s}{LC}$

For our series RLC, we found:

- $\alpha = \frac{R}{2L}$
- $\omega_0 = \frac{1}{\sqrt{LC}}$
- $\hat{f}(t) = \frac{v_s}{LC}$

We will have to obtain these values on a per-circuit basis by obtaining the governing differential equation.

Note. The general solution to our general form will follow:

$$x(t) = x_n(t) + x_f(t).$$

Where x_n is the natural solution that solve the equation for $\hat{f} \equiv 0$, involving two arbitrary constants. x_f is the forced, or particular, or steady-state response with no constants, which forms the complete response along with x_n. △

Let's solve our general form. First find the natural response with $\hat{f} \equiv 0$.

$$\ddot{x}_n + 2\alpha\dot{x}_n + \omega_0^2 x_n(t) = 0.$$

Take the ansatz $x_n(t) = e^{st} = \exp\left(\left(-\alpha \pm \sqrt{\alpha^2 - \omega_0^2}\right)t\right), s \in \mathbb{C}$. Subbing this into the DE will recover the characteristic equation, from which we obtain our expression for s in terms of the parameters of our equation. Since our solution is second-order linear, we can express the solution space as a linear combination of our two 'basis' solutions for s_1 and s_2.

$$x_n(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}.$$

We have three cases:

- $\alpha^2 > \omega_0^2$: this gives two distinct, real solution, resulting in a sum of two decaying exponentials, or an over damped solution.
- $\alpha = \omega_0$: we get a critically damped response of the form:

$$x_n(t) = (A_1 t + A_2)e^{st}.$$

- $\alpha^2 < \omega_0^2$: this gives two complex roots, corresponding with an under damped, or oscillatory solution.

Now we consider the particular solution, or the forced response.

$$\ddot{x}_f + 2\alpha\dot{x}_f + \omega_0^2 x_f = \hat{f}.$$

We assume that the source is constant, $\hat{f}(t) = \hat{f}$, and then using method of undetermined coefficients, we obtain $x_f = K = \frac{\hat{f}}{\omega_0^2}$.

Therefore, our general solution for $\alpha^2 \neq \omega_0^2$ is:

$$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + K.$$

And for $\alpha = \omega_0$:

$$x(t) = e^{st}(A_1 t + A_2) + K.$$

Lecture 9

2026-01-21

Let's examine our underdamped case. Take $\omega_n = \sqrt{\omega_0^2 - \alpha^2} > 0$ be the natural frequency. Our complex exponents become $s_{1,2} = -\alpha \pm j\omega_n$, to which we can apply Euler's identity and symmetries of sin and cos to obtain:

$$\begin{aligned} x_n(t) &= e^{-\alpha t}\left[(A_1 + A_2)\cos\omega_n t + j(A_1 - A_2)\sin\omega_n t\right] \\ &= e^{-\alpha t}\left[B_1 \cos\omega_n t + jB_2 \sin\omega_n t\right], \quad B_1, B_2 \in \mathbb{R}. \end{aligned}$$

This, graphically, manifests as a sinusoidal oscillation with an exponentially-decaying amplitude envelope.

Let's examine our critically damped case, which returns to equilibrium in the fastest time possible.

$$x_n(t) = (A_1 t + A_2)e^{-\alpha t}.$$

This solution crosses the 'steady-state' axis at most one time.

Let's examine our overdamped case, which has no oscillations.

$$x_n(t) = A_1 e^{\left(-\alpha + \sqrt{\alpha^2 - \omega_0^2}\right)t} + A_2 e^{\left(-\alpha - \sqrt{\alpha^2 - \omega_0^2}\right)t}.$$

This also crosses the 'steady-state' axis at most one time.

Given a second order circuit:

- Find the differential equation in standard form:

$$\ddot{x} + 2\alpha\dot{x} + \omega_0^2 x = \hat{f}(t);$$

- Extract the parameters α and ω_0 and characterize natural response;
- Write general solution $x(t) = x_n(t) + K$ for our natural response x_n;
- Find the arbitrary constants in the *general solution* using known conditions, obtaining the derivative condition by writing inductors at $t = 0^+$ as current sources and capacitors as voltage sources.

$$\begin{aligned} x\left(0^+\right) &= x_n\left(0^+\right) + K \\ \dot{x}\left(0^+\right) &= \dot{x}_n\left(0^+\right). \end{aligned}$$

2026-01-23 **Lecture 10**

We have studied the time-domain response of circuits with constant sources. For first order circuits, we found the canonical form of the solution as:

$$x(t) = x(\infty) + \left(x\left(0^{+}\right) - x(\infty)\right) e^{-\frac{t}{\tau}}.$$

For $t_0 = 0$, and $\tau = R_{eq}C, \tau = \frac{L}{R_{eq}}$.

For second order circuits, we found the canonical form of the differential equation to be:

$$\ddot{x} + 2\alpha\dot{x} + \omega_0^2 x = \hat{f}.$$

Resulting in an overdamped, critically damped, or underdamped natural response along with a particular solution, a forced response.

2 Complex Plane Analysis and Generalized Phasors

Recall that we can write an exponentially damped sinusoid as:

$$V_m e^{\sigma t}\cos(\omega t + \theta) = Re\left\{V_m e^{\sigma t} e^{j(\omega t + \theta)}\right\}, V_m, \sigma, \omega, \theta \in \mathbb{R}.$$

We can regroup our terms to combine the time-dependent exponents of complex frequency and amplitude decay, which will group our complex phase angle with the maximum amplitude.

$$= Re\left\{V_m e^{j\theta} e^{(\sigma + j\omega)t}\right\}.$$

Let $s = \sigma + j\omega \in \mathbb{C}$ be the complex frequency of the damped sinusoid (dimensions of T^{-1}). The σ characterizes the exponential growth or decay, and the ω characterizes the oscillatory behaviour.

$$= Re\left\{V_m e^{j\theta} e^{st}\right\}.$$

2026-01-26 **Lecture 11**

We can define the generalized phasor $\tilde{V} = V_m e^{j\theta}$ describing the magnitude and phase of the damped sinusoid. This leaves our time-dependent function as:

$$Re\left\{\tilde{V} e^{st}\right\}.$$

If we apply a damped sinusoid input to a linear circuit, the result will also be a damped sinusoid of the same s, i.e. frequency and decay rate. Take an input $v_s(t) = Re\left[\tilde{V}_s e^{st}\right]$, the output variables of a linear circuit will have a forced response of the form $x_f(t) = Re\left[\tilde{x}_f e^{st}\right]$. The two phasors may have different amplitudes and phases.

It is physical to apply only a real input to the system. However, given a linear circuit, we can apply the whole complex input and then take the real portion of the output for our final stage.

Lecture 12

2026-01-28

Substituting the phasors and damped complex exponential ansatz into a time-domain differential equation will yield the *forced response*.

As a review, in the time domain we:

- Use circuit relations ($v_R = iR, v_L = L\frac{di_L}{dt}, i_C = C\frac{dv_c}{dt}$) along with circuit analysis tools to determine a time domain DE;
- We substitute our phasors and ansatz' into the time domain DE to determine the phasor relations;
- We take the real part of our complex solution.

We can alternatively do *phasor analysis* on a circuit in the frequency domain. In the frequency domain:

- We will instead use impedance $\tilde{Z}$ (or admittance $\tilde{Y}$) relations.
  - R: $\tilde{V} = \tilde{Z}\tilde{I}$ or $\tilde{I} = \tilde{Y}(s)\tilde{V}$
- We will use circuit analysis laws to directly determine a phasor relationship
- We take the real part of our complex solution.

Lecture 13

2026-01-30

Using our phasor forms:

$$v(t) = \tilde{V}e^{st}, \quad \tilde{V} = V_m e^{j\theta}$$
$$i(t) = \tilde{I}e^{st}, \quad \tilde{I} = I_m e^{j\phi}.$$

We can derive impedance relationships for various circuit components. The admittance relationships can be found using a reciprocal.

- Inductors

  Using $v = L\frac{di}{dt}$, the time dependence cancels and we obtain:

  $$\tilde{V} = Ls\tilde{I} \implies \tilde{Z} = \frac{\tilde{V}}{\tilde{I}} = sL.$$

  We observe that since $s = j\omega$, the impedance lies along the positive complex axis.

- Capacitors

  Using $i = C\frac{dv}{dt}$, the time dependence cancels and we obtain:

  $$\tilde{I} = Cs\tilde{V} \implies \tilde{Z} = \frac{1}{sC}.$$

  We observe that the impedance lies along the negative complex axis.

- Resistors

  $$\tilde{Z} = R.$$

2.1 The complex-frequency plane

Definition 10. A transfer function describes the ratio of a desired forced output phasor to the input phasor.

If $\tilde{V}_s$ is the input, and $\tilde{X}_f$ represents the forced part of the desired output, then the transfer function is given by:

$$\tilde{H}(s) = \frac{\tilde{X}_f}{\tilde{V}}.$$

The transfer function is independent of initial conditions, but is rather fully defined by the element configuration and impedances in the circuit.

2026-02-02

Lecture 14

The transfer function $\tilde{H}(s)$ is a complex function of $s = \sigma + j\omega$, or a function over the s-plane.

Note. The s-plane represents the damping coefficient and oscillating frequency on the complex plane. Further along the $-x$ axis, i.e. the direction of decreasing σ, is larger damping. Further up the $+i$ axis, i.e. the direction of increasing ω, is more rapid oscillation. △

The transfer function has a magnitude $|\tilde{H}(s)|$ and phase angle at each point above the s-plane. Both can be plotted as surfaces over the s-plane.

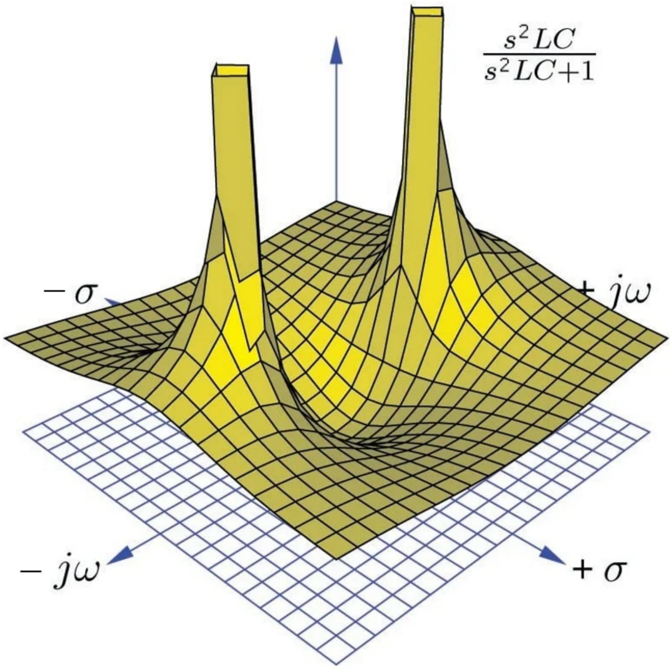

Figure 5: The magnitude of a given transfer function plotted over the s-plane

Example. Consider the transfer function:

$$\tilde{H}(s) = \frac{4(s+2)}{(s+3-j)(s+3+j)} = \frac{4(s-s_1)}{(s-s_2)(s-s_4)}.$$

Factors in the denominator are labelled as even s_i, and odd in numerator by convention. At s_1, the magnitude of the transfer function is 0, so s_1 is a 0 of the transfer function. As $s \to \infty$, the magnitude of the function goes to 0, so $s = \infty$ is also a 0. At s_2 and s_4, the transfer function goes to ∞, so s_2, s_4 are poles of the transfer function. $\Diamond$

Note. We choose to write transfer functions in the form:

$$\tilde{H}(s) = K\frac{(s-s_1)(s-s_3)\dots}{(s-s_2)(s-s_4)\dots}.$$

Where odd s_i are 0's of the transfer function where $|\tilde{H}(s) = 0|$, while even s_i are poles where $|\tilde{H}(s) \to \infty|$. $s \to \infty$ may also be a pole or zero. These are called the *critical frequencies* of the transfer function. $\triangle$

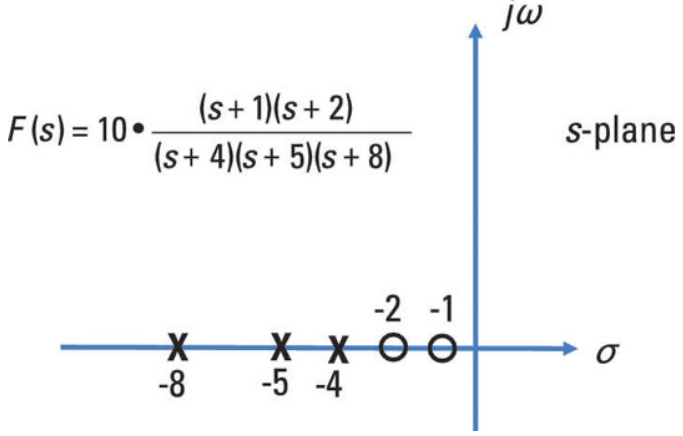

Figure 6: Here is a sample pole-zero constellation over the s-plane for the given transfer function. Open circles, O's, are zeros, and X's are poles.

We can plot the poles and zeros on a pole-zero constellation plot, which shows the location of critical frequencies on the s-plane. We can't really show behaviour as $s \to \infty$, but this can be noted on the plot as well.

Recall the series RLC circuit with source v_s and source phasor $\tilde{V}_s$. In the time domain, we obtained the differential equation:

$$\ddot{v}_c + \frac{R}{L}\dot{v}_c + \frac{1}{LC}v_c = \frac{v_s}{LC}.$$

And the characteristic equation in the frequency domain by subbing in a general phasor:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0.$$

In the frequency domain, we found our transfer function as:

$$\tilde{H}(s) = \frac{\tilde{V}_{cf}}{\tilde{V}_s} = \frac{1}{LC}\frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}}.$$

There is a symmetry between the characteristic equation and the poles of the transfer function. The roots of the characteristic equation are the poles of the transfer function.

2026-02-04
Lecture 15

Proof

The standard form of our second order circuit DE is:

$$\ddot{x} + 2\alpha\dot{x} + \omega_0^2 x = Gf\left(t\right).$$

The characteristic equation from the homogeneous equation $s^2 + 2\alpha s + \omega_0$ has the roots:

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}.$$

Now using $x_f = \tilde{X}_f e^{st}$, $f\left(t\right) = \tilde{F}e^{st}$ in the original DE (cancelling time-dependence):

$$\tilde{X}_f s^2 + 2\alpha\tilde{X}_f s + \omega_0^2\tilde{X}_f = G\tilde{F}.$$

We can then obtain the transfer function:

$$\tilde{H}\left(s\right) = \frac{\tilde{X}_f}{\tilde{F}} = G \cdot \frac{1}{s^2 + 2\alpha s + \omega_0^2}.$$

Which demonstrates that the poles are the roots of the characteristic equation.

Just as we can classify the form of the natural response by the roots of the characteristic equation, we can then deduce it from the poles of the transfer function. Let s_2, s_4 be the poles.

1. If $s_2 = \overline{s_4} = -\alpha \pm j\omega_n$ for $\omega_n = \sqrt{\omega_0^2 - \alpha^2}$, then the system is underdamped;
2. If $s_2 = s_4 = -\alpha \in \mathbb{R}$, then the system is critically damped;
3. If $s_2 \neq s_4 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}, s_2, s_4 \in \mathbb{R}$, then the system is overdamped.

Note. What if the order of the characteristic equation is 1?

$$\tilde{H}\left(s\right) = K \cdot \frac{1}{s - s_2}.$$

This represents a first order circuit, containing 1 pole. The natural response has the form:

$$x_n\left(t\right) = Ae^{s_2 t}.$$

Where $s_2 = -\frac{1}{\tau}$ is the negative reciprocal of the time constant. Prove this using the general DE to create the transfer function and find the poles! △

Why do the poles tell us about the natural response? This occurs when the input is 0, meaning $\tilde{H}\left(s\right) = \left|\frac{output}{input}\right| \to \infty$, and hence must coincide with the roots of the characteristic equation (the poles).

2026-02-06
Lecture 16

What do the zeros tell us about the circuit? Generally, zeroes indicate frequencies which are blocked by the circuit, i.e. the forced response will be zero irrespective of the magnitude and phase of the input (the phasor).

Lecture 17

2026-02-09

Procedure:

1. Find initial conditions for $t = 0^-, t = 0^+$, typically using tricks like continuity of functions and replacing inductors and capacitors with current and voltage sources;
2. For $t > 0$, draw the frequency domain circuit and find the transfer function $\tilde{H}(s)$;
3. Use the poles of $\tilde{H}(s)$ to write the form of the natural response $x_n(t)$;
4. Find the forced response using phasors and the transfer function $\tilde{X}_f = \tilde{H}(s)\tilde{V}_s$;
5. Write the complete time domain response by summing the natural and forced response and apply the initial conditions *to the complete response*;
6. Be careful with final answer units and domain restrictions!

2.2 Time Domain Response with Laplace Transforms

Suppose a single damped sinusoid will not suffice when describing a source. We can decompose a current or voltage into a sum of complex damped sinusoids such as the following:

$$\begin{aligned} y(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\tilde{Y}(\sigma + j\omega)\, e^{\sigma t}\left[\cos(\omega t) + j\sin(\omega t)\right]\right] d\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{Y}(\sigma + j\omega)\, e^{(\sigma + j\omega)t}\, d\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{Y}(s)\, e^{st}\, d\omega. \end{aligned}$$

This is an infinite, continuous sum of damped sinusoids, each with a specific magnitude and phase $\tilde{Y}(\sigma + j\omega)$. Note that σ is a constant chosen such that the integral converges. Any value of σ for which our integrals converge lies in the *region of convergence*. As such, $ds = j d\omega$, and we can rewrite:

$$y(t) = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty}\tilde{Y}(s)\, e^{st}\, ds.$$

For signals on $t > 0$, we can formally find $\tilde{Y}(s)$ as:

$$\tilde{Y}(s) = \int_{0^-}^{\infty} y(t)\, e^{-st}\, dt.$$

Which is the Laplace transform of our function $\tilde{Y}(s) = \mathcal{L}\{y(t)\}$. Note that our lower bound $t = 0^-$ embeds initial conditions or discontinuities.

A Laplace transform of a signal extracts the magnitude and phase information of a time-domain signal, related to an infinite sum of damped sinusoids. Given $\tilde{Y}(s)$, we can reconstruct the signal via the inverse Laplace transform.

Lecture 18

2026-02-11

A signal $y(t)$ captures information about a signal in the time domain, while its Laplace transform $\tilde{Y}(s)$ is a representation of a signal in the frequency domain. Since our circuits are linear, we can take Laplace transforms of time-domain variables and operate algebraically on them, regaining our time-domain signals at then end. Note that even if we include 0^- when finding the Laplace transform, we cannot use our $y(t)$ at $t = 0^-$. Our time-domain signals retrieved via an inverse Laplace transform are only valid for $t \geq 0^+$. This is visible in our equation for $\tilde{Y}(s)$, as it integrates $y(t)$ over 0^- to ∞. This allows it to capture initial discontinuities into the frequency spectrum, but does not capture information for $t < 0$.

When we track these transforms, we are tracking phasors, in terms of a general s. Linearity allows us to superimpose outputs for multiple values of s.

2026-02-25 **Lecture 19**

3 Frequency Response and Resonance

How does a forced output respond to changing the frequency of in input pure sinusoid? Key metrics are the gain $= \frac{B}{A}$ and the phase shift $= \phi - \theta$.

In the frequency domain, the *network transfer function* describes a system response to a pure sinusoid, $\tilde{H}(s) = \tilde{H}(j\omega)$.

$$\tilde{H}(j\omega) = \frac{Be^{j\phi}}{Ae^{j\theta}} = \frac{B}{A}e^{j(\phi-\theta)} = \text{gain}\angle\text{phase shift}.$$

When the gain is ≥ 1, then the circuit amplifies the input, while a gain of < 1 attenuates the input.

A filter is a network designed to amplify, pass, or attenuate signals depending on their frequency ranges.

- A passband is a range of frequencies allowed to pass (typically gain ≥ 1);
- A stopband is a range of frequencies that is blocked (gain < 1);
- Low-pass filter allows low frequencies to pass;
- High-pass filter allows high frequencies to pass;
- Band-pass filter allow a region of frequencies to pass.
- Band-reject filter rejects a region of frequencies

The cutoff frequency ω_c is defined as the frequency at which the output is at $\frac{1}{\sqrt{2}}$ of the maximum output value.

2026-02-26 **Lecture 20**

Definition 11. A bode plot is a visualization of a circuit's frequency response, displaying $|\tilde{H}(j\omega)|$ and $\angle\tilde{H}(j\omega)$ against frequency.

Conventions:

- Frequency $\omega = 2\pi f$ is done on a log scale;
- $|\tilde{H}(j\omega)|$ is plotted in dB: $20\log_{10}|\tilde{H}(j\omega)|$;

> **Note.** 20 is often used due to energy's square dependency on amplitude $P \propto A^2$, paired with log laws. If plotting power, we would use $10\log_{10}$. The extra order of magnitude stems from the deci- in decibels. △

For a simple RC low-pass filter, we can find the decibel magnitude of our transfer function to be:

$$H_{d\beta} = -20\log_{10}\sqrt{1+\frac{\omega}{\omega_c}}, \quad \omega_c = \frac{1}{RC}.$$

At the cut-off or $-3d\beta$ frequency, $\omega = \omega_c$, we get $H_{d\beta} \approx -3d\beta$. The magnitude of the output phasor at the cut-off frequency is $\frac{1}{\sqrt{2}}|\tilde{V}_{in}|$. The power $P \propto V^2$ has therefore fallen by $\frac{1}{2}$, so ω_c is also called the half-power frequency.

Lecture 21

2026-02-27

To build our phase plot, we note that $\angle\tilde{H}(j\omega) = -\tan\frac{\omega}{\omega_c}$. For $\omega \ll \omega_c \implies \angle\tilde{H}(j\omega) \approx 0$, and as ω grows, $\tilde{H}(j\omega) \to -90$. This corresponds with the idea that we examined the voltage across the capacitor as our output. For high frequency signals, the current leads the voltage by $90°$, whereas for low frequency signals, the impedance of the resistors dominates the circuit, resulting in an in-phase current and voltage.

In general, higher order circuits have a magnitude and phase expression that can be decomposed into sums of critical frequency logarithms and sums of arctan functions, respectively. This implies that each pole contributes an additional $-20d\beta$/decade to the magnitude spectrum for large frequencies, and each pole contributes an additional $-90°$ phase shift for high frequencies.

3.1 Resonance

Resonance is when the input frequency ω results in a circuit response approaching its maximum value.

> **Definition 12.** A network is in resonance when the voltage and current at the input terminals are in phase:
> $$\angle\tilde{V}_{in} = \angle\tilde{I}_{in}.$$
> The value of $\omega = \omega_0$ at which this occurs is called the resonant frequency.

Note that the phase equivalence implies an impedance 'seen' by the input signal that is completely real.

For a parallel RLC circuit, we obtain $\omega_0 = \frac{1}{\sqrt{LC}} \implies f_0 = \frac{1}{2\pi\sqrt{LC}}$.

Lecture 22

2026-03-02

What happens at resonance? For a parallel RLC circuit, the net impedance collapses to the impedance of the resistor. Therefore, the output voltage $\tilde{V}$ given a current source $\tilde{I}$ becomes $\tilde{V} = \tilde{I}R$, meaning the entire source current flows through the resistor.

However, despite the source current contained entirely in the resistor, the inductor and capacitor still have a voltage drop equivalent to $\tilde{V}$ since they are in parallel. We get: $\tilde{I}_L = \frac{\tilde{I}R}{j\omega_0 L}$ and

$\tilde{I}_C = \tilde{I} R j\omega_0 C$. Note that these currents are phase shifted by a relative 180°. This implies that the inductor and capacitor are 'exchanging' energy, or oscillating.

Since $w_L = \frac{1}{2}Li_L^2$ and $w_C = \frac{1}{2}Cv_c^2$, their power plots are half a period out of phase, and their sum is constant. Expanding their formulas:

$$|w_L(t)| = \frac{1}{2}L\left(\frac{v(t)}{\omega_0 L}\right)^2 = \frac{1}{2}\frac{v^2(t)}{\omega_0^2 L}$$
$$|w_C(t)| = \frac{1}{2}Cv^2(t).$$

Where we note that these functions have the same amplitude since $C = \omega_0^2 L$ in our circuit. However, they have relative phase shifts of $\frac{T}{2}$.

Definition 13. The quality factor describes the ratio between energy stored and energy lost in a circuit over a period, where Q_0 describes this value *at resonance*.

$$Q = 2\pi\left(\frac{\text{maximum energy stored in cct}}{\text{total energy lost in } T}\right).$$

2026-03-04

Lecture 23

Recall the response curve for our parallel RLC circuit, with a $\tilde{V}_{out}$ peak of $\tilde{I}R$ at $\omega_0 = \frac{1}{\sqrt{LC}}$. The critical frequencies, or half-power frequencies, bound the spike at $\frac{1}{\sqrt{2}}\tilde{V}_{out,max}$. We characterize the sharpness of our peak using these values.

Definition 14. The bandwidth is the difference between the two critical, or half-power frequencies:

$$B = \omega_H - \omega_L = \frac{w_0}{Q_0}.$$

Note that the resonant frequency is the geometric mean of the critical frequencies:

$$\omega_0 = \sqrt{\omega_L \omega_H}.$$

Note. As Q_0 gets larger (say ≥ 5), we can approximate our critical frequencies:

$$\omega_L \approx \omega_0\left(1 - \frac{1}{2Q_0}\right) = \omega_0 - \frac{1}{2}B, \quad \omega_R \approx \omega_0\left(1 + \frac{1}{2Q_0}\right) = \omega_0 + \frac{1}{2}B.$$

In this case, the bandwidth becomes the arithmetic mean of the critical frequencies. △

2026-03-05

Lecture 24

Imagine a series RLC circuit driven by a voltage source $\tilde{V}$. The net impedance is $R + j\left(\omega L - \frac{1}{\omega C}\right)$, and the transfer function is

$$\tilde{H}(j\omega) = \frac{\tilde{I}}{\tilde{V}} = \tilde{Y}.$$

By setting $\tilde{Z} = R$ at resonance, we obtain $\omega_0 = \frac{1}{\sqrt{LC}}$. Similarly, we can find the quality factor at resonance to be $Q_0 = \omega_0 \frac{L}{R}$.

Note. For a second-order under damped resonant circuit,the quality factor at resonance is $Q_0 = \frac{1}{2}\omega_0\tau$, where τ is the time constant. △

Further analysis reveals ω_L and ω_H match the parallel RLC results.

4 Two-Port Networks

Definition 15. A two port network is a circuit with two pairs of terminals, with each pair having the same current flowing in and out of each port's two terminals.

Note. A two-port network's circuit must be linear, and contain *no independent* sources. Additionally, source and loads must be connected only across the leads of a single port, *not* bridging between the two ports. △

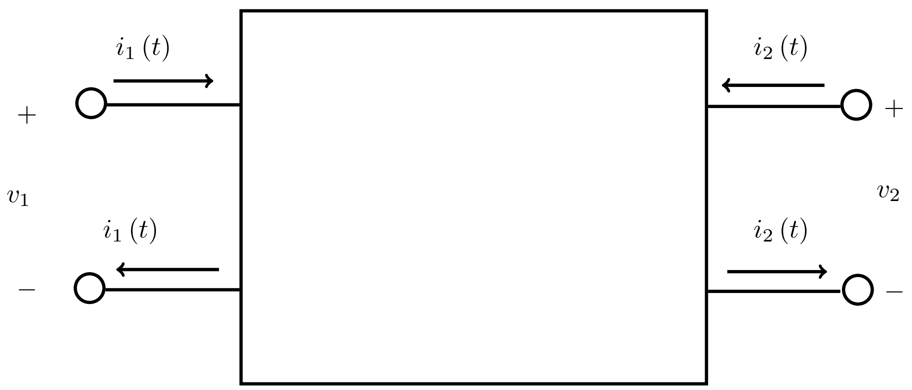

Figure 7: Diagram of a two port network. The generic port variables are phasors in the frequency domain, and functions in the time domain.

Lecture 25

2026-03-06

Since the circuits must be linear, and cannot contain any independent sources, then I_1 and I_2 are linear functions of V_1 and V_2.

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2.$$

Definition 16. The coefficients defining the current response y_{ij} are the *y-parameters* of the two-port.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}.$$

Where each admittance parameter y_{ij} can be found as:

$$y_{ij} = \frac{I_i}{V_j}|_{V_i=0}.$$

Each of these short circuit admittance parameters has a name:

- $y_{11} = \frac{I_1}{V_1}$ is called the short circuit input admittance;
- y_{12} and y_{21} are called short circuit transfer admittances;
- $y_{22} = \frac{I_2}{V_2}$ is called the short circuit output admittance.

Given the y-parameters, we have two linear equations in four circuit variables that will help us to solve a larger circuit incorporating the two-port.

2026-03-09

Lecture 26

Under certain instances, we can consider two two-ports, A and B, to be connected in parallel. One such instance occurs when:

- Each two port has a reference node that is common to its input and output, and the two reference nodes must be connected (common ground to each input and output);
- The non-reference input nodes must be connected, as well as the non-reference output nodes.

To find the y-parameters of the collective two-port, consider the individual parameters of A and B.

$$[I_A] = [y_A]\,[V_A]$$
$$[I_B] = [y_B]\,[V_B]\,.$$

By examining our circuit diagram, we note that $I = I_A + I_B$. We also note that the voltage across the input of our collective and individual two-ports are uniform: $V_1 = V_{A_1} = V_{B_1}$, similarly for the output. Therefore, we can sum our two matrix equations element-by-element, factoring out the common $[V]$ vector:

$$[I] = [y_A + y_B]\,[V]\,.$$

This is consistent with the notion that admittances sum directly for elements in parallel.

We can use impedance parameters instead of admittance parameters:

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2.$$

Definition 17. We can alternatively define a two-port using the *z-parameters*.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}.$$

Where each open-circuit impedance parameter (in Ohms) can be found as:

$$Z_{ij} = \frac{V_i}{I_j}|_{I_i=0}.$$

Note. The admittance parameters are related to the impedance parameters via an inverse:

$$[I] = [y]\,[V]\,, [V] = [z]\,[I] \implies [z] = [y]^{-1}\,.$$

△

Lecture 27

2026-03-11

Under certain instances, we can consider two two-ports, A and B, to be connected in series. One such instance occurs when:

- Each two port must have a common reference node stretching from input to output, and the two reference nodes must be connected (common ground);
- The networks are stacked one on top of the other;

To find the z-parameters of the collective two-port, consider the individual parameters of A and B.

$$\begin{aligned} [V_A] &= [z_A]\,[I_A] \\ [V_B] &= [z_B]\,[I_B]\,. \end{aligned}$$

By examining our circuit diagram, we note that $V = V_A + V_B$. We also note that the voltage across the input of our collective and individual two-ports are uniform: $I_1 = I_{A_1} = I_{B_1}$, similarly for the output. Therefore, we can sum our two matrix equations element-by-element, factoring out the common $[I]$ vector:

$$[V] = [z_A + z_B]\,[I]\,.$$

This is consistent with the notion that impedances sum directly for elements in series.

Definition 18. We can alternatively define a two-port using the *hybrid parameters*, which amounts to choosing two of the circuit variables and expressing them as a linear combination of the other two.

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}.$$

$$\begin{aligned} h_{11} &= \frac{V_1}{I_1}|_{V_2=0} && \text{short-circuit input impedance} \\ h_{12} &= \frac{V_1}{V_2}|_{I_1=0} && \text{open-circuit reverse voltage gain} \\ h_{21} &= \frac{I_2}{I_1}|_{V_2=0} && \text{short-circuit current gain} \\ h_{22} &= \frac{I_2}{V_2}|_{I_1=0} && \text{open-circuit output admittance .} \end{aligned}$$

Note. For reciprocal two ports, $h_{12} = -h_{21}$ (watch the negative!). Also, the hybrid parameters are often used to model transistors. △

2026-03-13 **Lecture 28**

Definition 19. We can define a two-port using the *transmission parameters*, which define the input parameters in terms of desired output parameters.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}.$$

$$\begin{aligned} t_{11} &= \frac{V_1}{V_2}|_{I_2=0} \\ t_{12} &= \frac{V_1}{-I_2}|_{V_2=0} \\ &\dots. \end{aligned}$$

Note. Both I_1 and $-I_2$ point in the direction of energy transmission. △

The transmission parameters are therefore useful for cascaded networks or transmission-line analysis. They are often called the $ABCD$ parameters.

Note. For reciprocal two-ports, $det\,[t] = t_{11}t_{22} - t_{12}t_{21} = 1$. △

Definition 20. For two two-ports in cascade, the net t-matrix will be the product of the two t-matrices of the corresponding individual two-ports:

$$[t] = [t_A]\,[t_B]\,.$$

2026-03-16 **Lecture 29**

5 Analysing Op-Amps

An operational amplifier is a combination of circuit elements that amplifies a differential input voltage at the output, which can be represented by a dependent voltage source of magnitude:

$$A\left(v_b - v_a\right) = Av_d.$$

Where A is the gain, v_b is the non-inverting input, v_a is the inverting input, and $v_d \equiv v_b - v_a$ is the differential input voltage. An op-amp requires a voltage input V^+ and V^- in order to power its amplification.

In an ideal operational amplifier:

$$R_i \to \infty, \quad R_o \to 0, \quad A \to \infty.$$

Therefore, in the ideal case we can assume that no current ever flows into the inverting and non-inverting input terminals ($i_b = i_a = 0$). Also, in the ideal case $R_o \to 0$, so at the output we assume that:

$$V_{out} = A\left(v_b - v_a\right) - R_o i_{out} = A\left(v_b - v_a\right) \implies v_b = v_a \text{ as } A \to \infty.$$

Note. Ideally, the voltages at the inverting and non-inverting inputs are equal. Additionally, the input currents are also assumed to be 0. △

An important feature of many op-amp systems is *negative feedback*, in which V_{out} is connected, in some form, to the *inverting* input v_a. This contributes to stability of the op-amp circuit, as increasing the input voltage differential $v_b > v_a$ results in feedback that increase v_a and restores equilibrium of our inputs. For example, using a feedback resistor R_f results in a gain of $A = -\frac{R_f}{R_i}$.

Lecture 30 2026-03-20

6 Diodes and Diode Circuits

A diode is the simplest non-linear circuit element that allows current to pass in only one direction.

For an ideal diode, when $v < 0$, the current is clamped to $i = 0$ and the diode can be treated as an open circuit. The diode is 'off' when it is reverse biased.

When $v > 0$, the diode is said to be forward-biased, or on, and ideally behaves like a short circuit.

Lecture 31 2026-03-23

To solve diode circuits:

1. First make an assumption about the state of each diode;
2. Then, analyse the circuit *consistent* with each diode assumption made;
3. Lastly, chech whether the results are self-consistent.
    - Diodes assumed to be on must have a zero forwards voltage and positive current, anode to cathode;

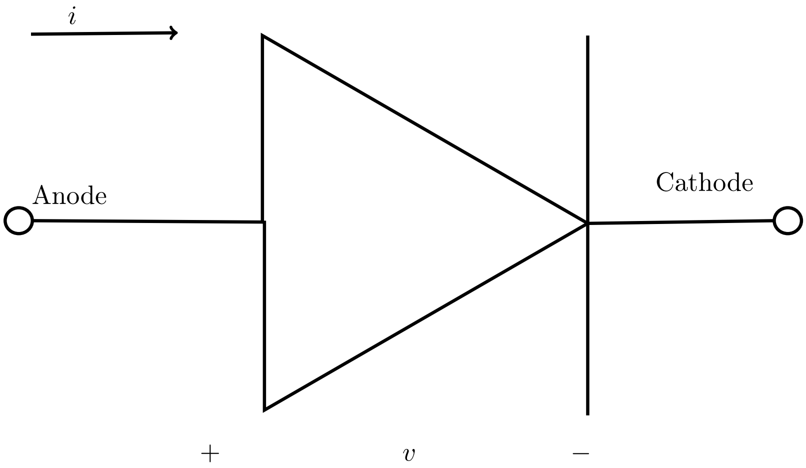

Figure 8: Diode circuit symbol

- Didoes assuemd to be off must have a negative forwards voltage and zero current.

If self-consistency is satisfied, then we are done!

2026-03-25
Lecture 32

A semiconductor junction diode is when a *pn junction* is created in silicon. A realistic diode has a characteristic curve with three distinct sections:

- For $v > 0$, we have forwards bias. This presents as a concave up, steeply increasing curve, which particularity upticks beyond the *forwards voltage* of the diode.
- For $v < 0$, we have the reverse bias region, at which fA of current pass backwards through the diode.
- For sufficiently large negative v, we have the breakdown region. Breakdown occurs at $-V_{zk}$.

Outside of the breakdown region, the current and voltage are related by:

$$i = I_s \left(e^{\frac{v}{nV_T}} - 1 \right).$$

Where i is the measured current forwards through the diode and v is the applied voltage. I_s is the magnitude of the saturation current backwards through the diode, which depends on temperature, is proportional to the diode area. $V_T = \frac{k_B T}{e}$ is the thermal voltage in terms of the Boltzmann constant k_B, the temperature $T\,(K)$, and the elementary charge. n is the ideality factor that lies between $1 \to 2$.

Note. Examine the forwards bias region assuming $n = 1$, $V_T = 25mV$, v is appreciable (so we can drop the -1), and $I_s \sim 10^{-15}A$.

$$i \approx I_s e^{\frac{v}{V_T}} = 10^{-15} e^{\frac{v}{V_T}}.$$

A voltage of $0.5V$ across the diode gives $i = 500nA$, which approaches the μA range and is therefore turning on. A voltage of $0.7V$ gives $i = 1mA$. Around this $0.7V$ mark, the current grows *very* rapidly. At this point, the diode is fully conducting, or working in its designed current range (for tens of mA diode). △

Note. If $v < 0$ and a few times nV_T, then $i \approx -I_s$ will be very small. In reality, $i \approx -I_s \cdot K$ for some scaling factor $K > 1$, however it is still very small! △

Note. The breakdown region occurs when $v < -V_{zk}$. △

Lecture 33

2026-03-27

We can describe the forwards bias region using the exponential model, which approximate the true curve:

$$i_D = I_s \left(e^{V_D/nV_T} - 1\right), \quad \text{true curve}$$
$$i_D = I_s e^{V_D/nV_T}, \quad \text{approximation for } v_D \geq 0.$$

This models the true curve very well for $v_D \geq 3nV_T$, however it is still non-linear.

Note. When solving diode circuit with this model, we will obtain a transcendental equation that must be solve graphically for the intersections, or solutions. The *Q-point*, or *quiescent point*, is the intersection, or solution. The linear constraint equation is called the *load line*, as it is the constraint placed upon the non-linear element by the load. △

We can also apply the battery-plus-resistance model. We assume 0 current for all $v < v_{D_0}$. v_{D_0} is the chosen diode operating voltage parameter. For larger voltages, we apply a linear fit with a slope $\frac{1}{r_D}$, where r_D is the diode resistance parameter. In general, we choose this to be very small to obtain a steep response curve. Our model becomes:

$$i_D = \begin{cases} 0 & v_D < v_{D_0} \\ \frac{v_D - v_{D_0}}{r_D} & v_D \geq v_{D_0} \end{cases}.$$

Our model circuit for our real diode (in forwards bias) is an ideal diode in series with a voltage source of magnitude v_{D_0} opposing current, and a resistor of value r_D.

A third model is the *constant voltage drop* model. We assume the same idea as above, but set $r_D = 0\Omega$. This idea assumes that a forwards conducting diode will exhibit a constant voltage drop of $v_{D_0} = 0.7V$, irregardless of the driving voltage. This can be analysed like an ideal diode, but simply ensuring that the forwards voltage is $> 0.7V$ if the diode is considered to be open.

6.1 Rectifier Circuits

A rectifier scircuit takes a purely AC signal and creates an output with a DC component.

Lecture 34

2026-03-30

A *half-wave rectifier* uses a diode in series with the source. The average value of an input sine wave v_s is 0, but the processed version of v_s, or the output v_0, has a non-zero average. This output is unidirectional, as it is 0 for a half cycle and approximately non-zero (due to forwards voltage of a real diode) for another half cycle.

2026-04-01

Lecture 35

A *full-wave rectifier* uses a sequence of four diodes in a 'diode bridge' to rectify an oscillating source voltage.

Consider real diodes with $V_{D_0} \neq 0$. When $-V_{D_0} < v_s < V_{D_0}$, the diodes are all off, and no current flows through the load. When $v_s > V_{D_0}$, two diodes are on and our net KVL is:

$$v_s - 2V_{D_0} = v_0.$$

This same analysis holds for $v_s < -V_{D_0}$.